TOPIC: TWO MEANS – MATCHED PAIRS (DEPENDENT SAMPLES)

Introduction to Matched Pairs

◆ Two data sets are **Matched Pairs** if they are __________ & each value is *paired* to another in a _____ relationship.

► Common Relationships: **1)** before & after of same individual; **2)** related individuals; **3)** self-reported vs measured.

EXAMPLE Determine if the samples are matched pairs. If so, calculate the difference ($d = x_1 - x_2$) between each matched pair. Then find the mean difference ($\bar{d}$) and standard deviation (s_d).

New **Matched Pairs**

(A) The data below shows heart rates from a sample of 9 adults before and after sleeping.

Heart Rates (beats per minute)									
Before	84	70	68	79	71	85	90	65	56
After	80	73	78	91	69	77	91	81	79
Diff. (d)*									

**ALWAYS subtract in the same order*

$n_1 = n_2$ ☐

Samples are related ☐

Values paired 1:1 ☐

(B) The data below shows heart rates from a sample of adult males and females.

Heart Rates (beats per minute)									
Males	84	70	68	79	71	85	90	65	56
Females	80	73	78	91	69	77	91	81	79
Diff. (d)*									

$n_1 = n_2$ ☐

Samples are related ☐

Values paired 1:1 ☐

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EXAMPLE

For the following scenarios, determine if the two samples are independent or matched pairs.

1) A nutritionist measures the blood pressure of 30 patients before and after they begin a new diet, with the goal of determining whether the diet has an effect on blood pressure.

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2) A teacher gives 40 students a practice exam, then a week later gives a different group of 40 students the same practice exam. The teacher wants to know the difference in scores.

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3) A teacher gives 40 students a practice exam, then a week later gives the same 40 students a similar graded exam. The teacher wants to know if practice improves scores.

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PRACTICE

A personal trainer is studying whether a new stretching routine improves flexibility. She records the forward reach (in cm) of 6 clients before and after a 4-week program. Calculate the difference (after – before) for each client, the mean difference, and standard deviation.

Stretching Reach (cm)		
Client	Before	After
A	22	26
B	19	23
C	24	27
D	21	20
E	18	18
F	23	25

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Matched Pairs: Hypothesis Tests

◆ When conducting Hyp. Tests using matched pairs, replace $\bar{x}, \mu, s$ with __________.

EXAMPLE

A company claims its new medication reduces blood pressure. In a study, the sample mean difference in 37 patients' blood pressure before and 15 days after taking the medication was 3.2 with a standard deviation of 6.7. Perform a hypothesis test using $\boldsymbol{\alpha = 0.05}$ to determine if the medication is effective in reducing blood pressure.

Random samples? ☐

Matched pairs? ☐

Normal ***OR*** $n > 30$? ☐

1) H_0:

H_a:

$t =$

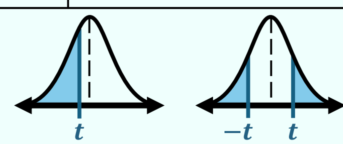
2) $n =$ ______

$\bar{d} =$ ______

$s_d =$ ______

3) $df =$

P-value:

	1 Mean	Matched Pairs
Hyp.	$H_0: \mu = \#$ $H_a: \mu$ </>/≠ #	$H_0: \mu_d = \#$ $H_a: \mu_d$ **[< | > | ≠]** #
Test Stat.	$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$	$t = \dfrac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$ $n =$ # of pairs
P-Val.	Area "beyond" t $df = n - 1$	
Conclude	Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0. There is **[ENOUGH | NOT ENOUGH]** evidence to { restate H_a }	

4) Because P-value **[< | >]** α, we **[REJECT | FAIL TO REJECT]** H_0.

There is **[ENOUGH | NOT ENOUGH]** evidence that the medication is effective in reducing blood pressure.

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EXAMPLE

A professor randomly selects 12 students and records their scores on a pop quiz and a scheduled quiz given in the same week. Each student's two scores are paired, and the difference is calculated as $(\text{Pop Quiz Score} - \text{Scheduled Quiz Score})$. From the data, she calculates $\bar{d} = -1.1$, $s_d = 1.8$. Write H_0 & H_a to test the claim that students perform worse on pop quizzes than scheduled quizzes, and calculate the test statistic.

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Difference in Means: Confidence Intervals

◆ To make a confidence interval for μ_d use point estimator $\bar{d}$ and margin of error:

New

$E = t_{\alpha/2} \cdot \frac{}{\sqrt{n}}$

EXAMPLE

In a study on the effectiveness of a new blood pressure medication, the sample mean difference in 37 patients' blood pressure before and 15 days after taking the medication was 3.2 with a standard deviation of 6.7. Make a 90% confidence interval for the mean difference in blood pressure. What does this result suggest about the claim that there is no difference after taking the medication?

We are ______ % confident that the true difference in blood pressure before and after the medication is between ________ & ________. Because this **[DOES | DOES NOT]** include 0, we **[REJECT | FAIL TO REJECT]** $H_0: \mu_d = 0$. There is **[ENOUGH | NOT ENOUGH]** evidence that there is a difference...

HOW TO: Make a Confidence Interval for $p_1 - p_2$

1) Verify samples are:

Matched Pairs **AND** Random ☐

Normal ***OR*** $n > 30$ ☐

2) Find critical value: $t_{\alpha/2}$

3) Point estimate: $\bar{d}$

4) Margin of Error: $E = t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}$

5) Find upper & lower bounds

$$(\bar{d} - E, \bar{d} + E)$$

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PRACTICE

Construct a 95% confidence interval for the mean difference of the population given the following information. Would you reject or fail to reject the claim that there is no difference in the mean?

$\bar{d} = -0.728$

$s_d = 1.34$

$n = 10$