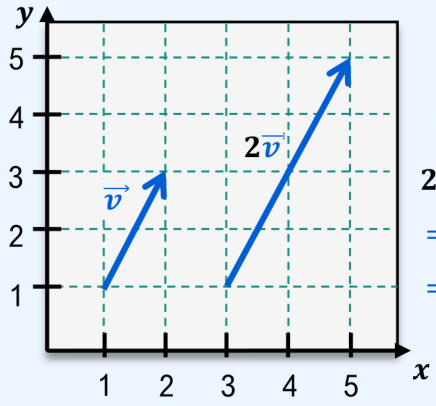
TOPIC: DOT PRODUCT

Introduction to Dot Product

◆ Dot Product: A way to "multiply" vectors. To calculate, _____ "like" components, then _____ the values.

- Generally represents how close the vectors are to pointing in the *same* direction.

Recall — Product of Scalar & Vector



$\vec{v} = \langle 1, 2 \rangle$

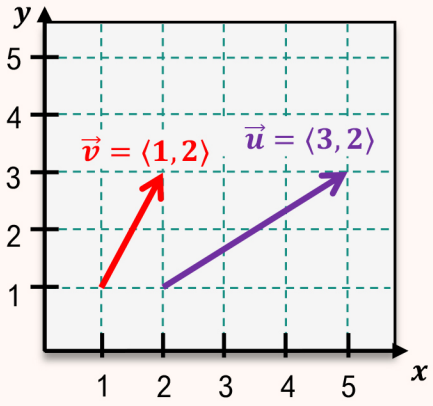
$2\vec{v} = 2 \cdot \langle 1, 2 \rangle$

$= \langle 2 \cdot 1, 2 \cdot 2 \rangle$

$= \langle 2, 4 \rangle$

$c\vec{v}$ = [SCALAR | VECTOR]

New — (Dot) Product of Vector & Vector



$\vec{v} \cdot \vec{u} = v_x \cdot \_ + v_y \cdot \_$

$\vec{u} \cdot \vec{v}$ = [SCALAR | VECTOR]

EXAMPLE Complete the following vector operations below.

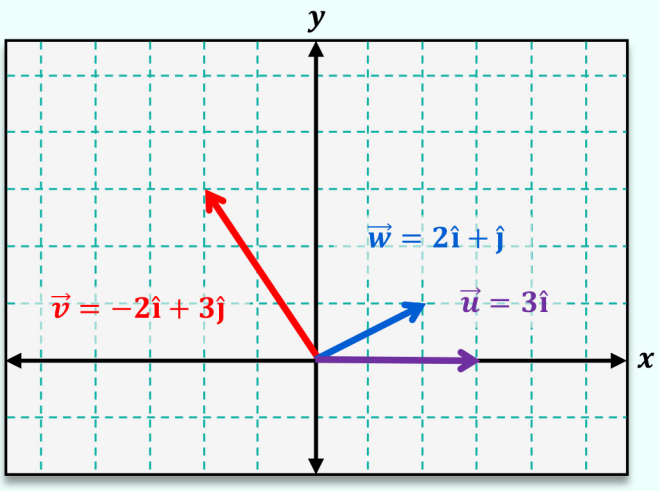
(A) $\vec{u} \cdot \vec{w}$

_____

(B) $\vec{v} \cdot \vec{w}$

_____

(C) $\vec{u} \cdot (\vec{w} + \vec{v})$



◆ When the dot product between two vectors is ____, the vectors are perpendicular.

TOPIC: DOT PRODUCT

PRACTICE If vectors $\vec{v} = \langle 4,3 \rangle$ and $\vec{u} = \langle 9,1 \rangle$, calculate $\vec{v} \cdot \vec{u}$.

PRACTICE If vectors $\vec{v} = 12\hat{\imath}$ and $\vec{u} = 100\hat{\jmath}$, calculate $\vec{u} \cdot \vec{v}$.

PRACTICE If vectors $\vec{a} = 13\hat{\imath}$, $\vec{b} = 5\hat{\imath} - 12\hat{\jmath}$, and $\vec{c} = 24\hat{\jmath}$, calculate $\vec{b} \cdot (\vec{a} - \vec{c})$.

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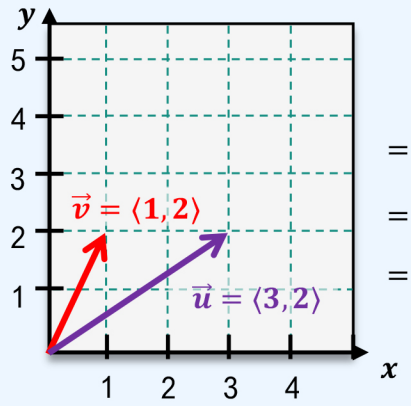
Find the Angle Between Two Vectors

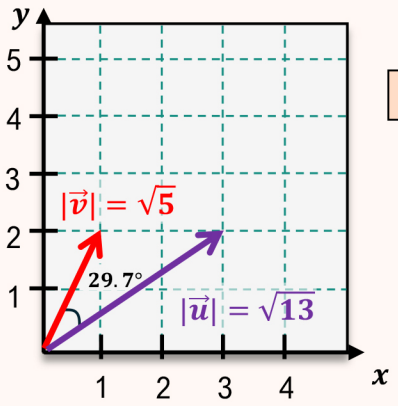
◆ The dot product can be found using the magnitudes and _____ between two vectors.

Recall **Dot Product – Using Components**



$$\vec{v} \cdot \vec{u}$$
$$= \langle 1, 2 \rangle \cdot \langle 3, 2 \rangle$$
$$= 1 \cdot 3 + 2 \cdot 2$$
$$= 7$$

New **Dot Product – Using Magnitudes & ∠**



$$\vec{v} \cdot \vec{u} = |\vec{v}||\vec{u}| \cdot \_\_\_\_\_$$

◆ To find the smallest angle between 2 vectors, rearrange the dot product formula & solve for θ.

EXAMPLE If the dot product for vectors $|\vec{a}| = 4$ and $|\vec{b}| = 8$ is: $\vec{a} \cdot \vec{b} = 16$, find the *angle* between vectors $\vec{a}$ & $\vec{b}$.

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PRACTICE If vectors $|\vec{a}| = 3$ and $|\vec{b}| = 7$, and $\vec{a} \cdot \vec{b} = 14.85$, determine the angle between vectors $\vec{a}$ & $\vec{b}$.

PRACTICE If vectors $\vec{a} = 4\hat{\imath}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath}$ determine the angle between vectors $\vec{a}$ & $\vec{b}$.

PRACTICE If vectors $|\vec{v}| = 12$, $|\vec{u}| = 100$ and the angle between $\vec{v}$ & $\vec{u}$ is $\theta = \frac{\pi}{6}$, calculate $\vec{v} \cdot \vec{u}$.