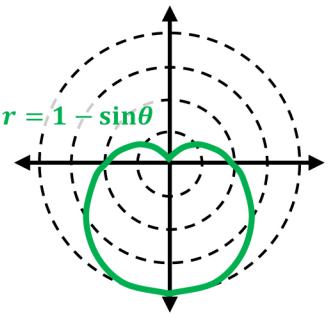
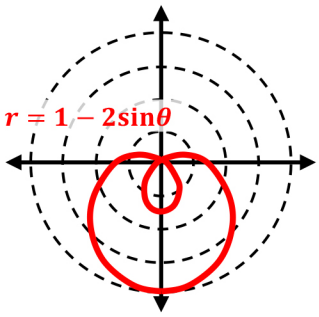
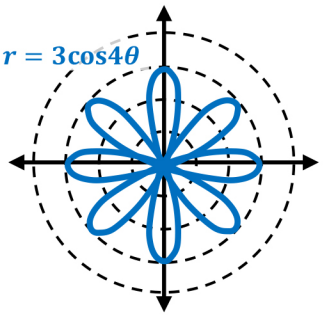
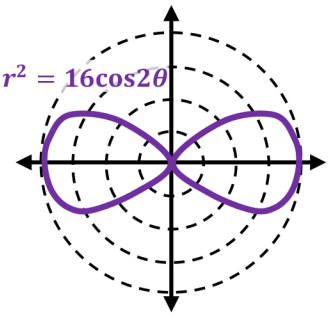
TOPIC: GRAPHING OTHER COMMON POLAR EQUATIONS

Introduction to Common Polar Equations

◆ When working with polar equations, you need to *classify* and *graph* these four shapes:

Cardioids	Limaçons	Roses	Lemniscates
			
$r = a \pm b\cos\theta$ $r = a \pm b\sin\theta$	$r = a \pm b\cos\theta$ $r = a \pm b\sin\theta$	$r = a\cos\_\_\theta$ $r = a\sin\_\_\theta$	$r \quad = \pm a^2\cos 2\theta$ $r \quad = \pm a^2\sin 2\theta$
$a > 0, b > 0,$ a ____ b	$a > 0, b > 0,$ a ____ b OR a ____ b *(dimple)* *(inner loop)*	$a \neq 0$ $n = 2, 3, 4 \ldots$	$a \neq 0$

EXAMPLE Classify the given equation.

(A) $r = 1 + \cos\theta$

[CARDIOID | LIMACON | ROSE | LEMNISCATE]

(B) $r = 4\sin 2\theta$

[CARDIOID | LIMACON | ROSE | LEMNISCATE]

PRACTICE Identify whether the given equation is that of a cardioid, limaçon, rose, or lemniscate.

(A) $r = 4\sin 2\theta$ **[CARDIOID | LIMACON | ROSE | LEMNISCATE]**

(B) $r = 3 + 2\cos\theta$ **[CARDIOID | LIMACON | ROSE | LEMNISCATE]**

(C) $r^2 = -25\cos 2\theta$ **[CARDIOID | LIMACON | ROSE | LEMNISCATE]**

(D) $r = 1 - \sin\theta$ **[CARDIOID | LIMACON | ROSE | LEMNISCATE]**

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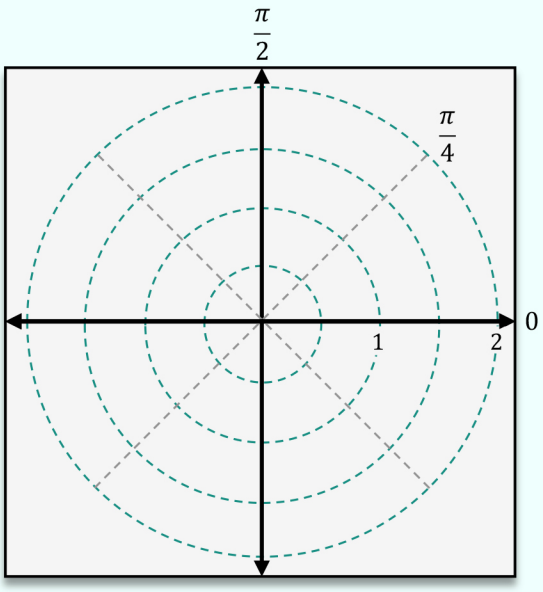
Cardioids

- Recall: A cardioid is of the form: $r = a \pm b\cos\theta$ OR $r = a \pm b\sin\theta$ where $a = b$.
  - To graph a cardioid, first determine **symmetry**, then plot **points** at quadrantal angles.

EXAMPLE Graph $r = 1 + \cos\theta$



HOW TO: Graph a Cardioid

1) **Symmetry** → If eqn contains:

cos	sin
polar axis	$\theta = \frac{\pi}{2}$

2) Find & plot **points** at quadrantal angles

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r				

3) **Connect** with smooth, continuous curve

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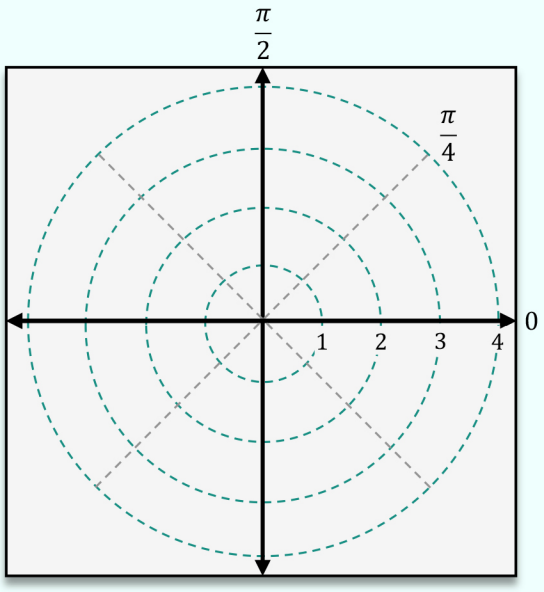
EXAMPLE Graph $r = 2 + 2\sin\theta$



HOW TO: Graph a Cardioid

1) **Symmetry** → If eqn contains:

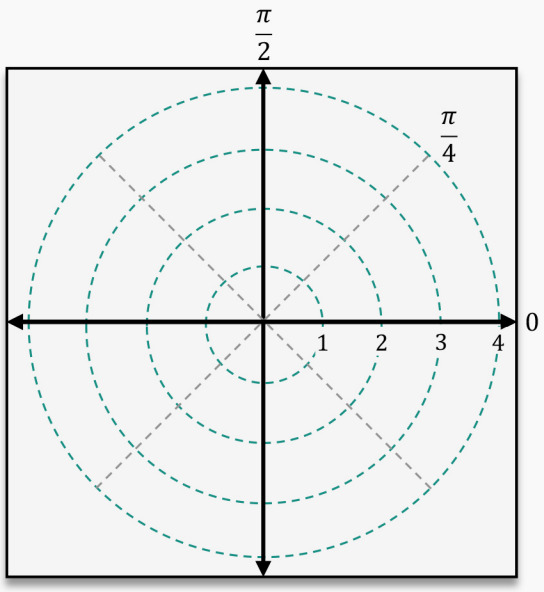
cos	sin
polar axis	$\theta = \frac{\pi}{2}$

2) Find & plot **points** at quadrantal angles

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r				

3) **Connect** with smooth, continuous curve

PRACTICE Graph $r = 2 - 2\cos\theta$



HOW TO: Graph a Cardioid

1) **Symmetry** → If eqn contains:

cos	sin
polar axis	$\theta = \frac{\pi}{2}$

2) Find & plot **points** at quadrantal angles

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r				

3) **Connect** with smooth, continuous curve

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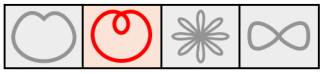
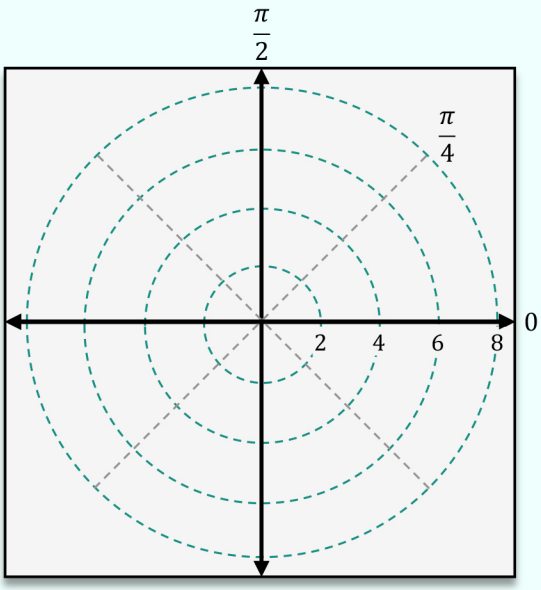


Limaçons

◆ Recall: A limaçon is of the form: $r = a \pm b\cos\theta$ OR $r = a \pm b\sin\theta$ where $a > b$ OR $a < b$.

- Graph limaçons just like cardioids, but first determine if it has a **dimple** or **inner loop**.

EXAMPLE Graph $r = 3 - 2\sin\theta$



HOW TO: Graph a Limaçon

1) If a [> | <] b, limaçon has [dimple / no zero | inner loop / zero]

2) Symmetry → If eqn contains: [cos / polar axis | sin / $\theta = \frac{\pi}{2}$]

3) Find & plot **points** at quadrantal angles

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r				

4) **Connect** with smooth, continuous curve

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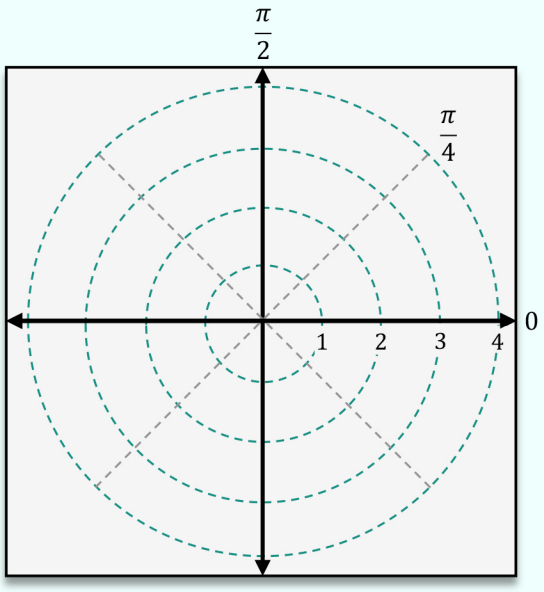
EXAMPLE Graph $r = 1 + 2\cos\theta$



HOW TO: Graph a Limaçon

1) If a [> | <] b, limaçon has [**dimple** / **no zero** | **inner loop** / **zero**]

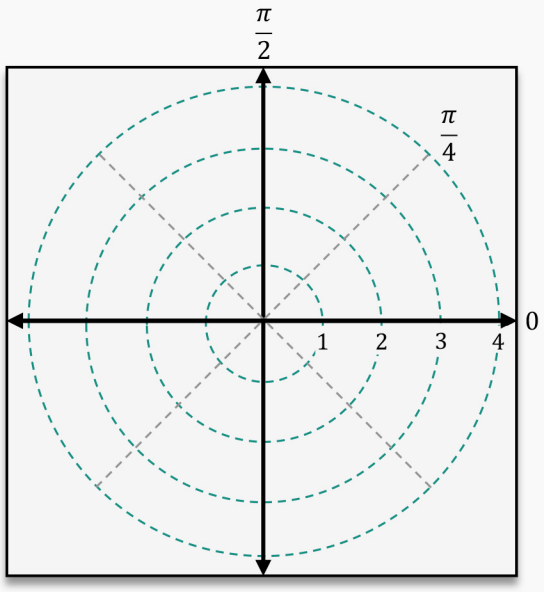
2) **Symmetry** → If eqn contains: [cos / polar axis | sin / $\theta = \frac{\pi}{2}$]

3) Find & plot **points** at quadrantal angles

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r				

4) **Connect** with smooth, continuous curve

PRACTICE Graph $r = 1 + 2\sin\theta$



HOW TO: Graph a Limaçon

1) If a [> | <] b, limaçon has [**dimple** / **no zero** | **inner loop** / **zero**]

2) **Symmetry** → If eqn contains: [cos / polar axis | sin / $\theta = \frac{\pi}{2}$]

3) Find & plot **points** at quadrantal angles

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r				

4) **Connect** with smooth, continuous curve

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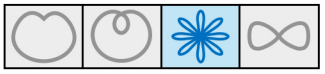
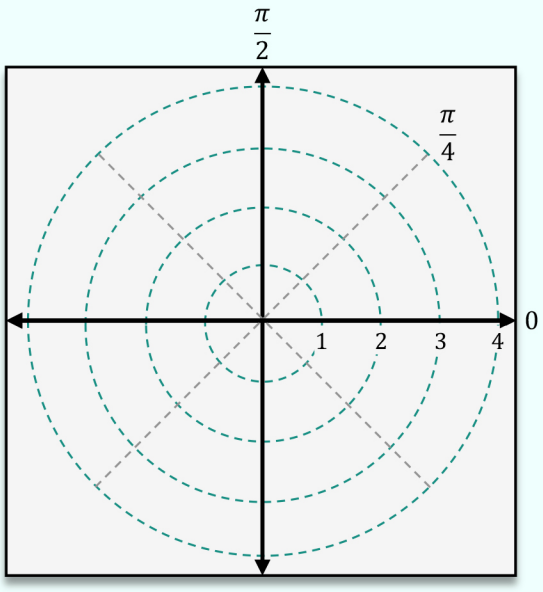


Roses

◆ Recall: A rose is of the form: $r = a\cos n\theta$ OR $r = a\sin n\theta$ where $a \neq 0, n \geq 2$.

- To graph a rose, plot **first petal**, then determine the spacing of other **petals**.

EXAMPLE Graph $r = 4\cos 2\theta$



HOW TO: Graph a Rose

1) For n [**even** | **odd**], [$2n$ | n] **petals**: _____

2) Plot **first petal**: (_____,_____)
$\boldsymbol{a}$

cos	sin
$\theta = 0$	$\theta = \frac{\pi}{2n}$

3) Plot other **petals** spaced out by $\frac{\boldsymbol{2\pi}}{\textbf{\# petals}}$: $\frac{\boldsymbol{2\pi}}{\textbf{\# petals}}$

$\boldsymbol{\theta}$				
$\boldsymbol{r}$				

4) **Connect** with smooth, continuous curve

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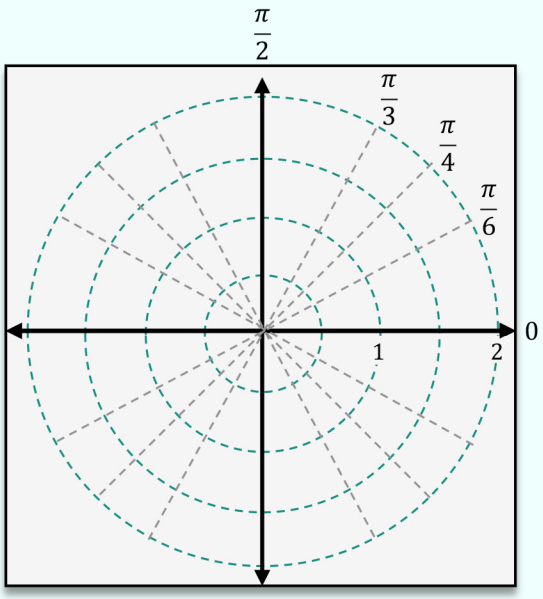
EXAMPLE Graph $r = 2\sin 3\theta$



HOW TO: Graph a Rose

1) For n [even | odd], [$2n$ | n] petals: _____

2) Plot first petal: (____,____)
a

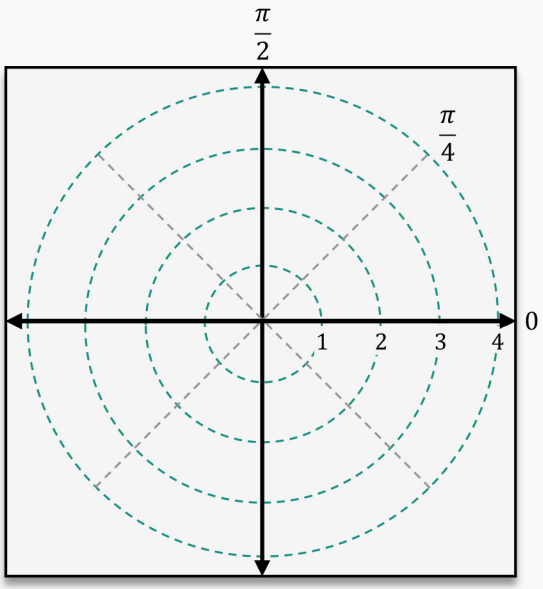
$\cos$	$\sin$
$\theta = 0$	$\theta = \frac{\pi}{2n}$

3) Plot other petals spaced out by $\frac{2\pi}{\#\text{ petals}}$: $\frac{2\pi}{\quad}$

θ			
r			

4) **Connect** with smooth, continuous curve

PRACTICE Graph $r = 3\cos 4\theta$



HOW TO: Graph a Rose

1) For n [even | odd], [$2n$ | n] petals: _____

2) Plot first petal: (____,____)
a

$\cos$	$\sin$
$\theta = 0$	$\theta = \frac{\pi}{2n}$

3) Plot other petals spaced out by $\frac{2\pi}{\#\text{ petals}}$: $\frac{2\pi}{\quad}$

θ								
r								

4) **Connect** with smooth, continuous curve

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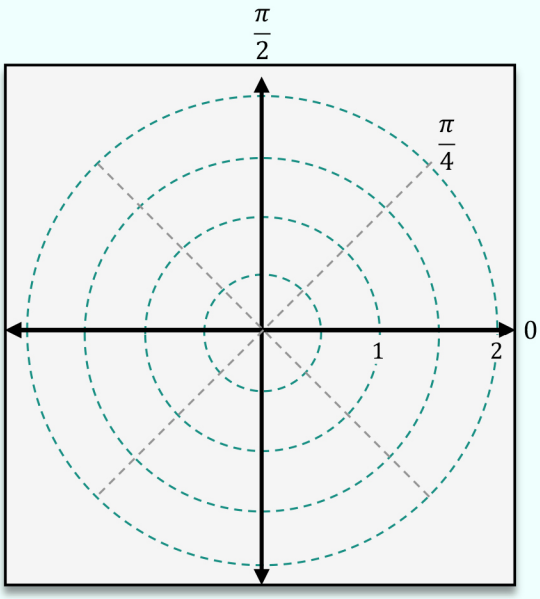
Lemniscates

- Recall: A lemniscate is of the form: $r^2 = \pm a^2 \cos 2\theta$ OR $r^2 = \pm a^2 \sin 2\theta$ where $a \neq 0$.
  - To graph a lemniscate, plot the **first petal**, then reflect it over the pole.

EXAMPLE Graph $r^2 = 4 \sin 2\theta$



HOW TO: Graph a Lemniscate

1) Plot **first petal**: (____,____)
a

$+a^2\cos$	$-a^2\cos$	$+a^2\sin$	$-a^2\sin$
$\theta = 0$	$\theta = \frac{\pi}{2}$	$\theta = \frac{\pi}{4}$	$\theta = \frac{3\pi}{4}$

2) Reflect **petal** over pole

3) **Connect** with smooth, continuous curve

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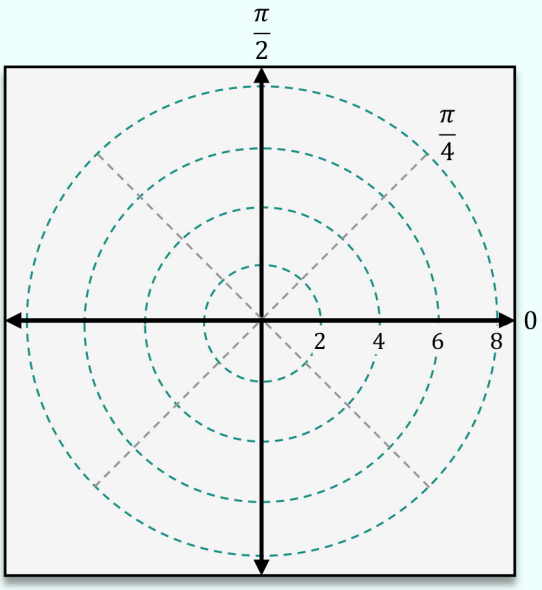
EXAMPLE Graph $r^2 = 25\cos 2\theta$



HOW TO: Graph a Lemniscate

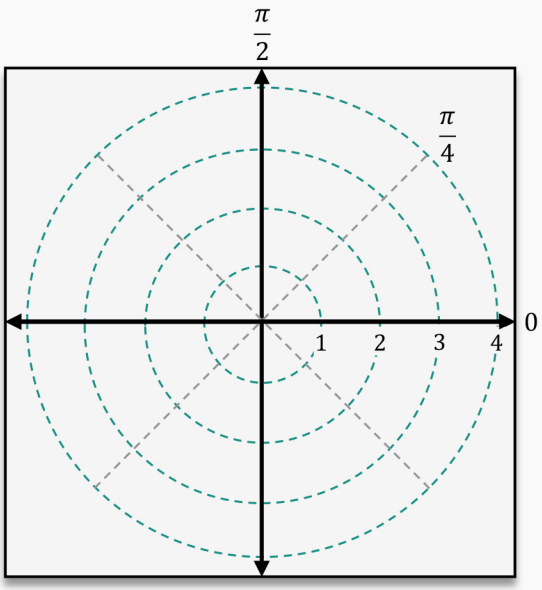
1) Plot **first petal**: (____,____)

$+a^2\cos$	$-a^2\cos$	$+a^2\sin$	$-a^2\sin$
$\theta = 0$	$\theta = \frac{\pi}{2}$	$\theta = \frac{\pi}{4}$	$\theta = \frac{3\pi}{4}$

2) Reflect **petal** over pole

3) **Connect** with smooth, continuous curve

PRACTICE Graph $r^2 = 9\sin 2\theta$



HOW TO: Graph a Lemniscate

1) Plot **first petal**: (____,____)

$+a^2\cos$	$-a^2\cos$	$+a^2\sin$	$-a^2\sin$
$\theta = 0$	$\theta = \frac{\pi}{2}$	$\theta = \frac{\pi}{4}$	$\theta = \frac{3\pi}{4}$

2) Reflect **petal** over pole

3) **Connect** with smooth, continuous curve