# MASTER TABLE: TRIG IDENTITIES

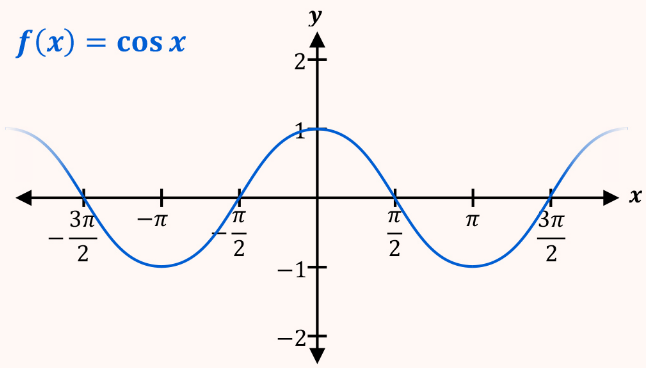
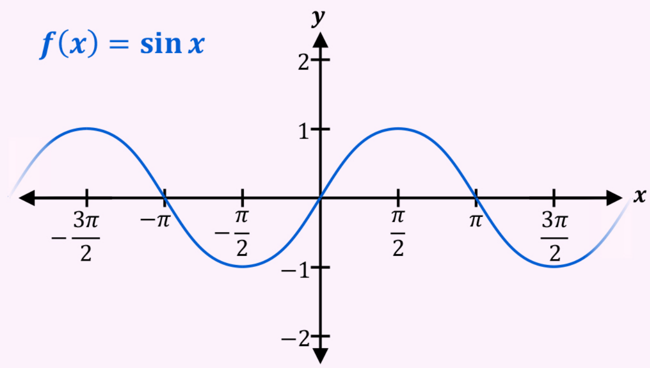
◆ **NOTE**: This table spans multiple videos.

| TRIG IDENTITIES | | | |
|---|---|---|---|
| **Name** | **Identity** | **Example** | **Use when...** |
| Reciprocal | $\csc\theta = \frac{1}{\sin\theta}$ | $\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\left(\frac{1}{2}\right)} = 2$ | You need to rewrite an expression in terms of $\sin$ & $\cos$ |
| | $\sec\theta = \frac{1}{\cos\theta}$ | | |
| | $\cot\theta = \frac{1}{\tan\theta}$ | | |
| Quotient | $\tan\theta = \frac{\sin\theta}{\cos\theta}$ | $\tan\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)}$ | |
| | $\cot\theta = \frac{\cos\theta}{\sin\theta}$ | | |
| Even – Odd | $\cos(-\theta) = \_\_\_\cos\theta$ | $\cos\left(-\frac{\pi}{4}\right) =$ | *argument* is ______________. |
| | $\sin(-\theta) = \_\_\_\sin\theta$ | | |
| | $\tan(-\theta) = \_\_\_\tan\theta$ | $\csc\left(\frac{\pi}{6}\right) =$ | |
| Pythagorean | $\sin^2\theta + \cos^2\theta = 1$ | $\sin^2\frac{11\pi}{6} + \cos^2\frac{11\pi}{6} =$ | you see trig functions _________. |
| | $\_\_\_\_\theta + \_\_\_\_\_ = \_\_\_\_\theta$ | | |
| | $\_\_\_\_\_ + \_\_\_\_\theta = \_\_\_\_\theta$ | | |
| Sum & Diff. | $\sin(a \pm b) = \_\_\_\_a\ \_\_\_\_\_b \pm \_\_\_\_\_a\ \_\_\_\_\_b$ | $\sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) =$ | argument contains a _____/_____ *OR* multiples of $15°$ or $\frac{\pi}{12}$ |
| | $\cos(a \pm b) = \_\_\_\_a\ \_\_\_\_b \mp \_\_\_\_\_a\ \_\_\_\_\_b$ | | |
| | $\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$ | | |
| Double Angle | $\sin 2\theta = 2\_\_\_\_\_\_\_\_\_\_\_\_\_\_$ | $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12} =$ | argument contains ______ *OR* you recognize a _______ of the identity |
| | $\cos 2\theta = \cos^2\theta - \sin^2\theta$<br>$= 1 - 2\sin^2\theta$<br>$= 2\cos^2\theta - 1$ | | |
| | $\tan 2\theta = \frac{2\tan\theta}{1 - \_\_\_\_\_^2\,\theta}$ | | |

## TOPIC: INTRODUCTION TO TRIG IDENTITIES

### Even & Odd Identities

◆ If you know a function is **even** or **odd**, you can easily find $f$( ______ ).

| Even Function | Odd Function |
|---|---|
|   |   |
| $f(-x) =$ [ $f(x)$ \| $-f(x)$ ] | $f(-x) =$ [ $f(x)$ \| $-f(x)$ ] |
| Symmetric on [ $y$ – axis \| ORIGIN ] | Symmetric on [ $y$ – axis \| ORIGIN ] |
| Trig Functions: [ cos sec \| sin csc \| tan cot ] | Trig Functions: [ cos sec \| sin csc \| tan cot ] |

◆ An **identity** is an equation which is TRUE for ________ possible values.

**TRIG IDENTITIES**

| Name | Identity | Example | Use when... |
|---|---|---|---|
| Even – Odd | $\cos(-\theta) =$ ____ $\cos\theta$ | $\cos\left(-\frac{\pi}{4}\right) =$ | *argument* is ____________. |
| | $\sin(-\theta) =$ ____ $\sin\theta$ | $\csc\left(-\frac{\pi}{6}\right) =$ | |
| | $\tan(-\theta) =$ ____ $\tan\theta$ | | |

## TOPIC: INTRODUCTION TO TRIG IDENTITIES

**EXAMPLE** Use the even/odd identities to rewrite the expression with no negative arguments in terms of one trig function.

**(A)** $-\tan(-\theta)$

**(B)** $\dfrac{\sin(-\theta)}{\cos(-\theta)}$

**Recall**

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

*(Even – Odd Identities)*

**PRACTICE** Use the even-odd identities to evaluate the expression.

**(A)** $\cos(-\theta) - \cos\theta$

**(B)** $-\cot(\theta) \cdot \sin(-\theta)$

## TOPIC: INTRODUCTION TO TRIG IDENTITIES

**PRACTICE** Use the even-odd identities to evaluate the expression.

**Recall**

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

*(Even – Odd Identities)*

**(A)**

$\sec\left(-\frac{4\pi}{5}\right)$

$\cos\frac{4\pi}{5}$ $\quad -\cos\frac{4\pi}{5}$ $\quad \sec\frac{4\pi}{5}$ $\quad -\sec\frac{4\pi}{5}$

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**(B)**

$\sin(-38°)$

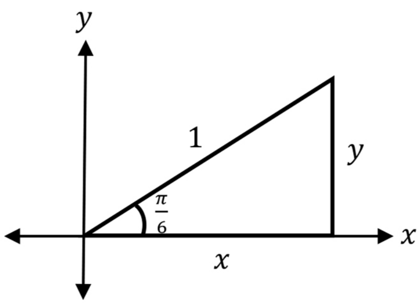
$\sin 38°$ $\quad -\sin 38°$ $\quad -\sin(-38°)$ $\quad \frac{1}{-\sin 38°}$

## TOPIC: INTRODUCTION TO TRIG IDENTITIES

### Pythagorean Identities

◆ You'll need the **Pythagorean Identities** to simplify expressions with ______________ trig functions.

- These identities come from combining the Pythagorean Theorem with the Unit Circle.




$$a^2 + b^2 = c^2$$

$$y^2 + x^2 = 1$$

**TRIG IDENTITIES**

| Name | Identity | Example | Use when… |
|---|---|---|---|
| Pythagorean | $\sin^2\theta + \cos^2\theta = 1$ | $\sin^2\frac{11\pi}{6} + \cos^2\frac{11\pi}{6} =$ | you see trig functions **squared**. |
| | $\_\_\_\_\_\theta + \_\_\_\_ = \_\_\_\_\_\theta$ | | |
| | $\_\_\_\_\_\_ + \_\_\_\_\theta = \_\_\_\_\_\theta$ | | |

◆ To rewrite trig expressions, you'll need to recognize different __________ of the Pythagorean Identities.

**EXAMPLE** Use the Pythagorean Identities to rewrite the expression as a single term.

**(A)**

$\sec^2\theta - \tan^2\theta$

**(B)**

$(1 - \cos\theta)(1 + \cos\theta)$

## TOPIC: INTRODUCTION TO TRIG IDENTITIES

**EXAMPLE** Use the Pythagorean Identities to rewrite the expression as a single term.

$$\frac{1}{1+\cos\theta}$$

**Recall**

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

(*Even – Odd Identities*)

**PRACTICE** Use the Pythagorean Identities to rewrite the expression as a single term.

$$(1+\csc\theta)(1-\csc\theta)$$

**PRACTICE** Use the Pythagorean Identities to rewrite the expression with no fraction.

$$\frac{1}{1-\sec\theta}$$

## TOPIC: INTRODUCTION TO TRIG IDENTITIES

### Simplifying Trig Expressions

◆ You'll need to use ALL trig identities to *fully* simplify expressions.

**EXAMPLE** Simplify the expression.

**(A)** $\tan(-\theta) \cdot \csc\theta$

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**(B)** $\dfrac{\sin^2\theta}{1+\cos\theta}$

**HOW TO: Fully Simplify Trig Expressions**

Trig expressions are fully simplified when…

- ▢ all arguments are ____________
- ▢ expression contains NO _____________
- ▢ expression contains as few trig fcns as possible

Strategies:

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of ______ & ______
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

**Recall** **Fundamental Trig Identities**

Reciprocal & Quotient

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Even/Odd

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Pythagorean

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

## TOPIC: INTRODUCTION TO TRIG IDENTITIES

**EXAMPLE** Simplify the expression.

**(A)**

$$\frac{\sin^2\theta - \tan^2\theta}{\sin\theta + \tan\theta}$$

---

**(B)**

$$\frac{\cos\theta + \csc\theta}{\cos\theta} + \frac{\sin\theta - \sec\theta}{\sin\theta}$$

**HOW TO: Fully Simplify Trig Expressions**

Trig expressions are fully simplified when…

- ▢ all arguments are positive
- ▢ expression contains NO fractions
- ▢ expression contains as few trig fcns as possible

Strategies:

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
- ◆ Break down in terms of sin & cos
- ◆ If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- ◆ Factor

**Recall** **Fundamental Trig Identities**

Reciprocal & Quotient

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

| Even/Odd | Pythagorean |
|---|---|
| $\sin(-\theta) = -\sin\theta$ | $\sin^2\theta + \cos^2\theta = 1$ |
| $\cos(-\theta) = \cos\theta$ | $\tan^2\theta + 1 = \sec^2\theta$ |
| $\tan(-\theta) = -\tan\theta$ | $1 + \cot^2\theta = \csc^2\theta$ |

## TOPIC: INTRODUCTION TO TRIG IDENTITIES

**PRACTICE** Simplify the expression.

**(A)**

$\tan^2\theta - \sec^2\theta + 1$

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**(B)**

$$\frac{\tan(-\theta)}{\sec(-\theta)}$$

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**(C)**

$$\left(\frac{\tan^2\theta}{\sin^2\theta} - 1\right)\csc^2\theta\cos^2(-\theta)$$

**HOW TO: Fully Simplify Trig Expressions**

Trig expressions are fully simplified when…

- ▢ all arguments are positive
- ▢ expression contains NO fractions
- ▢ expression contains as few trig fcns as possible

Strategies:

- ◆ Constantly scan for identities
- ◆ Add fractions using a common denominator
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**Recall** **Fundamental Trig Identities**

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$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

| Even/Odd | Pythagorean |
| --- | --- |
| $\sin(-\theta) = -\sin\theta$ | $\sin^2\theta + \cos^2\theta = 1$ |
| $\cos(-\theta) = \cos\theta$ | $\tan^2\theta + 1 = \sec^2\theta$ |
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## TOPIC: INTRODUCTION TO TRIG IDENTITIES

### Verifying Trig Equations as Identities

- To verify that an equation is true, simplify ONE or BOTH sides with the goal of making them __________.
  - ALWAYS start with the more ______________ side first!

**EXAMPLE** Verify the identity.

**(A)**

$$\frac{\sin\theta\cos\theta}{1-\cos^2\theta} = \frac{1}{\tan\theta}$$

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**(B)**

$$\frac{\sec^2\theta - \tan^2\theta}{\cos(-\theta)+1} = \frac{1-\cos\theta}{\sin^2\theta}$$

**STRATEGIES: Simplifying Trig Expressions**

- Constantly scan for identities
- Add fractions using a common denominator
- Break down in terms of sin & cos
- If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- Factor

**Recall** **Fundamental Trig Identities**

Reciprocal & Quotient

$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

| Even/Odd | Pythagorean |
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| $\sin(-\theta) = -\sin\theta$ | $\sin^2\theta + \cos^2\theta = 1$ |
| $\cos(-\theta) = \cos\theta$ | $\tan^2\theta + 1 = \sec^2\theta$ |
| $\tan(-\theta) = -\tan\theta$ | $1 + \cot^2\theta = \csc^2\theta$ |

# TOPIC: INTRODUCTION TO TRIG IDENTITIES

**EXAMPLE** Verify the identity by working with one side.

$$\frac{1-\sin\theta}{\cos\theta}-\frac{\cos\theta}{1+\sin\theta}=0$$

**STRATEGIES: Simplifying Trig Expressions**

- Constantly scan for identities
- Add fractions using a common denominator
- Break down in terms of sin & cos
- If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- Factor

**Recall** **Fundamental Trig Identities**

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$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

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## TOPIC: INTRODUCTION TO TRIG IDENTITIES

**EXAMPLE** Verify the identity by working with both sides.

$$\sec\theta\,(1-\sin^2\theta) = \frac{(1+\tan^2\theta)\cot^2\theta}{\csc^2\theta\sec\theta}$$

**STRATEGIES: Simplifying Trig Expressions**

- Constantly scan for identities
- Add fractions using a common denominator
- Break down in terms of sin & cos
- If $1 \pm \text{trig}(\theta)$, multiply top & bottom by $1 \mp \text{trig}(\theta)$
- Factor

**Recall** **Fundamental Trig Identities**

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## TOPIC: INTRODUCTION TO TRIG IDENTITIES

**PRACTICE** Identify the most helpful first step in verifying the identity.

**(A)**

$$\left(\frac{\tan^2\theta}{\sin^2\theta} - 1\right) = \sec^2\theta\,\sin^2(-\theta)$$

**(B)**

$$\sec^3\theta = \sec\theta + \frac{\tan^2\theta}{\cos\theta}$$

**STRATEGIES: Simplifying Trig Expressions**

- Constantly scan for identities
- Add fractions using a common denominator
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**Recall** **Fundamental Trig Identities**

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